Automatic Omniscope Calibration Using Redundant

SwFRT, Fermilab, 2009

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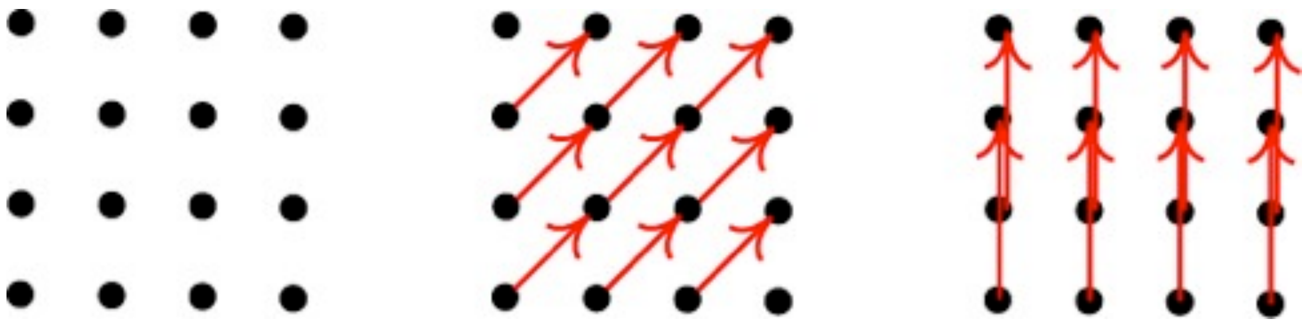
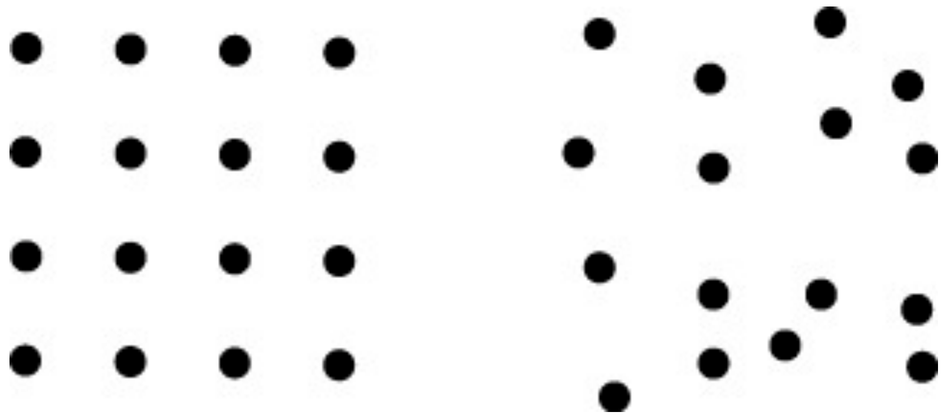
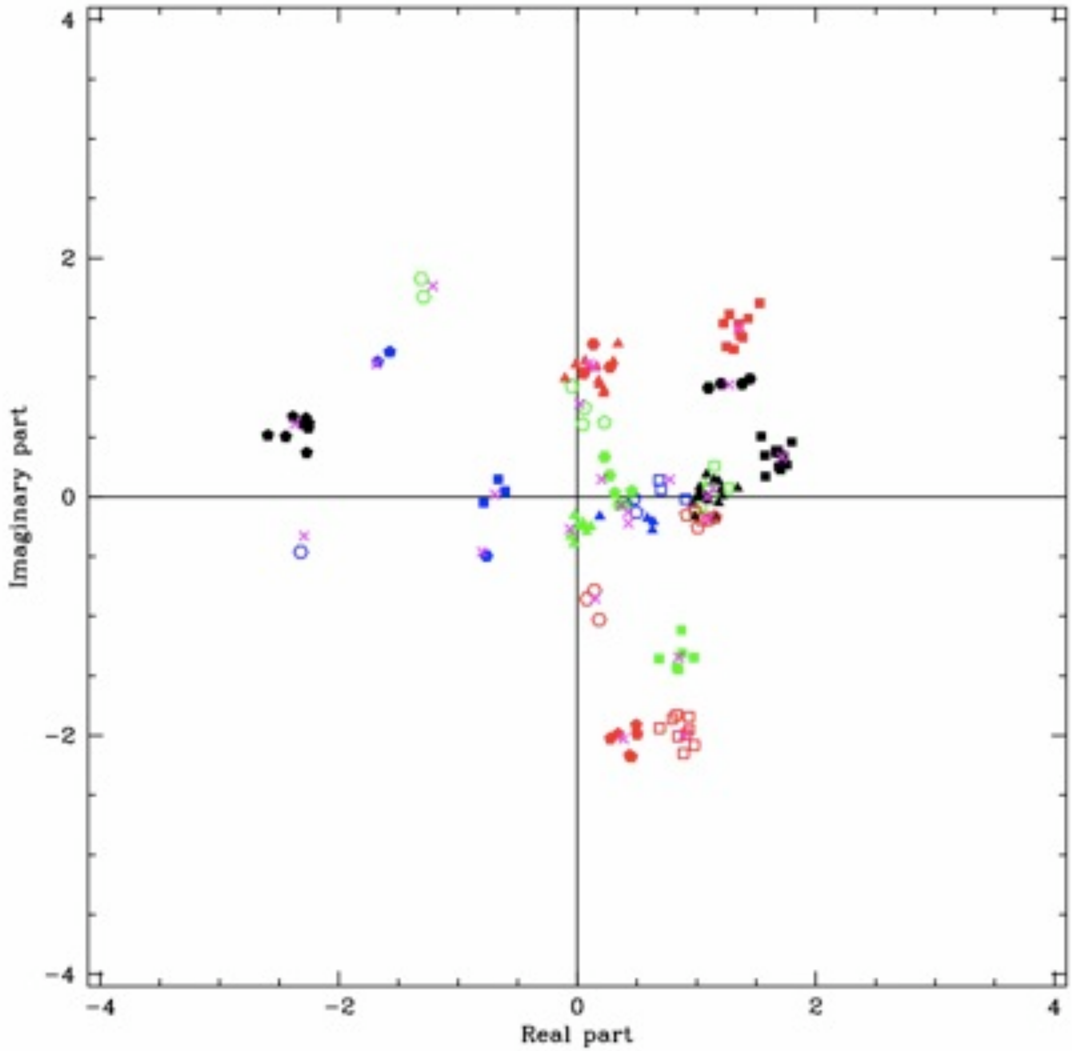
Unlike Traditional

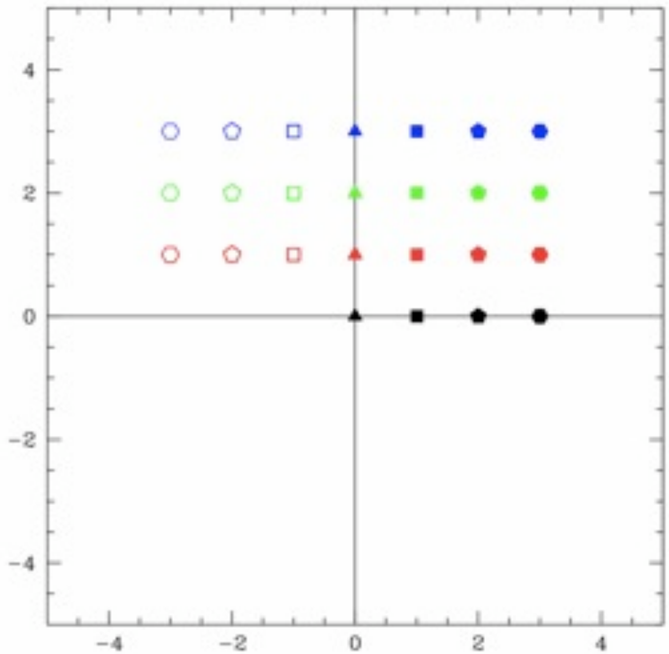


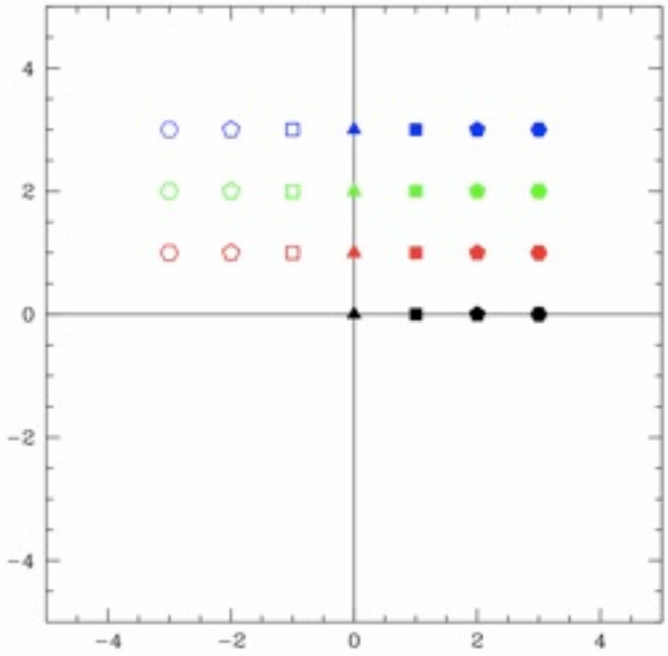
In Reality, Identical Baselines

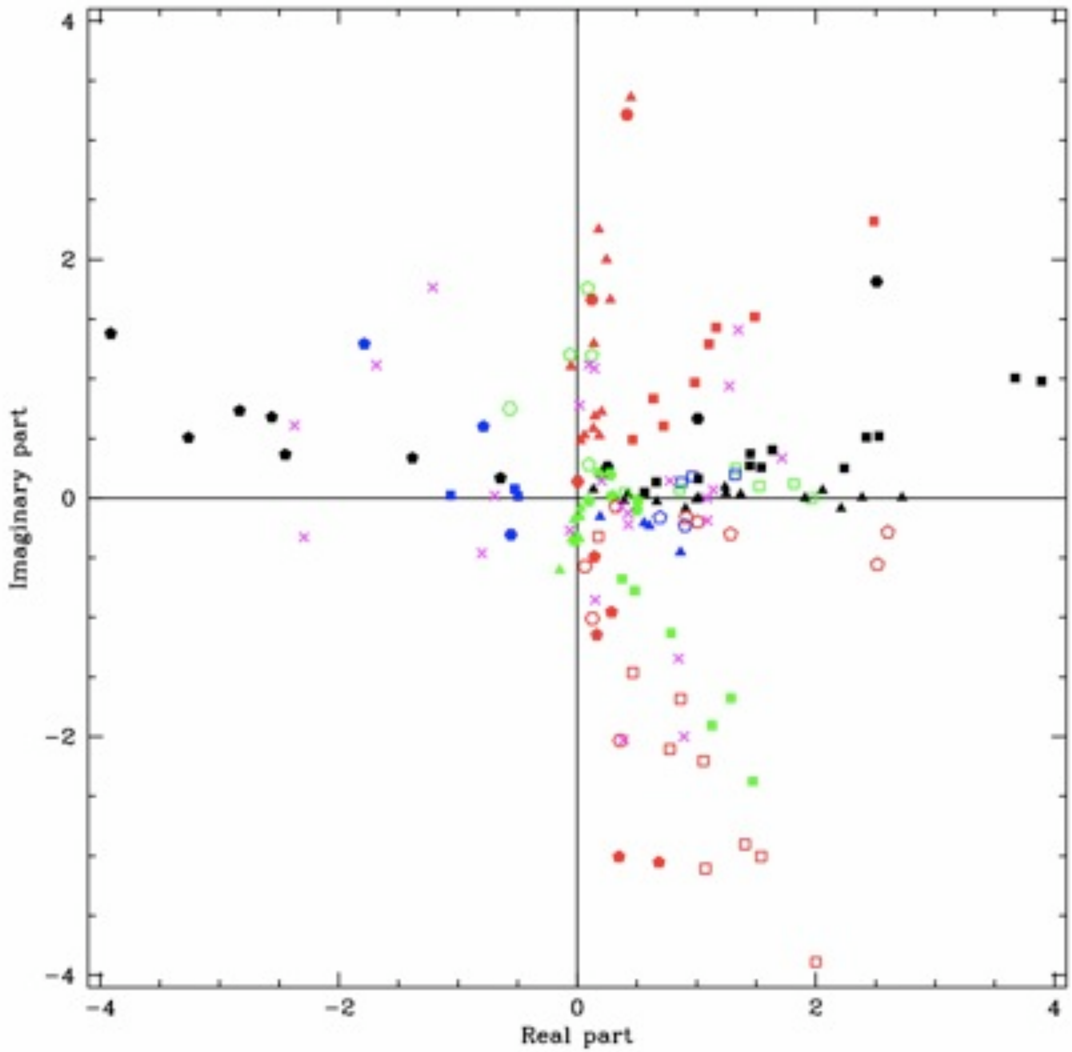


Main Idea: **<u>Demand</u>** that redundant baselines give the same results and fit





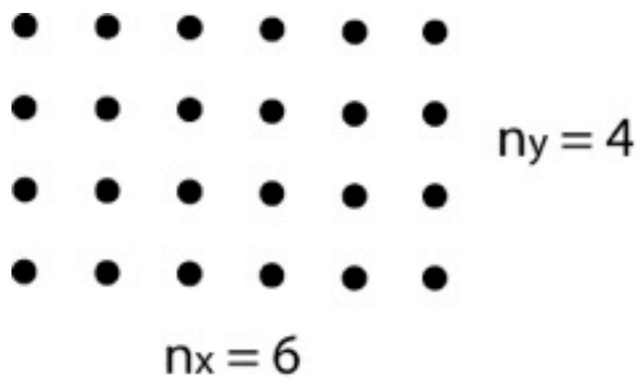




Imaginary part
Real part

Mathematically, an over-determined system

Inputs	Parameters being fit
• $n_x n_y (n_x n_y + 1)/2$ complex correlations	• $n_x n_y$ complex gains • $2\, n_x n_y - n_x - n_y + 1$ unique baselines



Measured signal

True sky signal

Instrumental noise

$$s_i(t) = g_i x_i(t) + n_i(t)$$

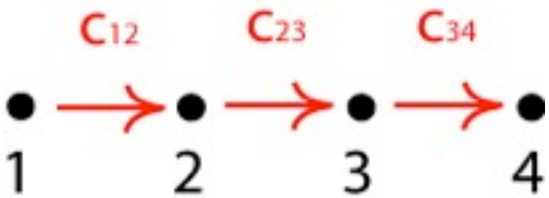
Complex gain: $g_i \equiv e^{\eta_i + i\varphi_i}$

$$\begin{aligned} c_{ij}(t) &\equiv s_i^*(t) s_j(t) \\ &= g_i^* g_j x_i^*(t) x_j(t) + \underbrace{n_i^*(t) n_j(t) + x_i^*(t) n_j(t) + x_j^*(t) n_i(t)}_{\text{Assume zero mean}} \end{aligned}$$

$$\langle c_{ij} \rangle = g_i^* g_j y_{i-j}(t)$$ where $$y_{i-j}(t) \equiv \langle x_i^*(t) x_j(t) \rangle$$

s_i = signal measured by antenna i ; g_i = gain of antenna i ; $g_i \equiv e^{\eta_i + i\varphi_i}$

corr; c_{ij} = measured corr



$$
\begin{aligned}
\langle c_{12}\rangle &= g_1^* g_2 y_1 \\
\langle c_{23}\rangle &= g_2^* g_3 y_1 \\
\langle c_{34}\rangle &= g_3^* g_4 y_1 \\
&\vdots \\
\langle c_{14}\rangle &= g_1^* g_4 y_3
\end{aligned}
$$

$$
\begin{aligned}
\ln\langle c_{12}\rangle &= (\eta_1 + \eta_2 + \ln|y_1|) + i(-\varphi_1 + \varphi_2 + \arg y_1) \\
\ln\langle c_{23}\rangle &= (\eta_2 + \eta_3 + \ln|y_1|) + i(-\varphi_2 + \varphi_3 + \arg y_1) \\
\ln\langle c_{34}\rangle &= (\eta_1 + \eta_2 + \ln|y_1|) + i(-\varphi_1 + \varphi_2 + \arg y_1) \\
&\vdots \\
\ln\langle c_{14}\rangle &= (\eta_1 + \eta_4 + \ln|y_3|) + i(-\varphi_1 + \varphi_4 + \arg y_3)
\end{aligned}
$$

s_i = signal measured by antenna i ; g_i = gain of antenna i

; $g_i \equiv e^{\eta_i + i\varphi_i}$ corr; c_{ij} = measured corr

Real Part: Gains

$$\begin{pmatrix} \mathrm{Re}\ln\langle c_{11}\rangle \\ \mathrm{Re}\ln\langle c_{22}\rangle \\ \mathrm{Re}\ln\langle c_{33}\rangle \\ \mathrm{Re}\ln\langle c_{44}\rangle \\ \mathrm{Re}\ln\langle c_{12}\rangle \\ \mathrm{Re}\ln\langle c_{23}\rangle \\ \mathrm{Re}\ln\langle c_{34}\rangle \\ \mathrm{Re}\ln\langle c_{13}\rangle \\ \mathrm{Re}\ln\langle c_{24}\rangle \\ \mathrm{Re}\ln\langle c_{14}\rangle \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \ln|y_0| \\ \ln|y_1| \\ \ln|y_2| \\ \ln|y_3| \end{pmatrix}$$

Have this (data)

Know this

Want this

s_i = signal measured by antenna i ; g_i = gain of antenna i ; $g_i \equiv e^{\eta_i + i\varphi_i}$ corr; c_{ij} = measured corr

Real Part: Gains

$$
\underbrace{\begin{pmatrix} \mathrm{Re}\ln\langle c_{11}\rangle \\ \mathrm{Re}\ln\langle c_{22}\rangle \\ \mathrm{Re}\ln\langle c_{33}\rangle \\ \mathrm{Re}\ln\langle c_{44}\rangle \\ \mathrm{Re}\ln\langle c_{12}\rangle \\ \mathrm{Re}\ln\langle c_{23}\rangle \\ \mathrm{Re}\ln\langle c_{34}\rangle \\ \mathrm{Re}\ln\langle c_{13}\rangle \\ \mathrm{Re}\ln\langle c_{24}\rangle \\ \mathrm{Re}\ln\langle c_{14}\rangle \end{pmatrix}}_{d} = \underbrace{\begin{pmatrix} 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \ln|y_0| \\ \ln|y_1| \\ \ln|y_2| \\ \ln|y_3| \end{pmatrix}}_{b}
$$

$$\hat{b} = [A^t A]^{-1} A^t \vec{d}$$

s_i = signal measured by antenna i ; g_i = gain of antenna i

; $g_i \equiv e^{\eta_i + i\varphi_i}$ corr; c_{ij} = measured corr

A Problem With Degeneracies

$$\hat{b} = [A^t A]^{-1} A^t \vec{d}$$

Real part (gains):

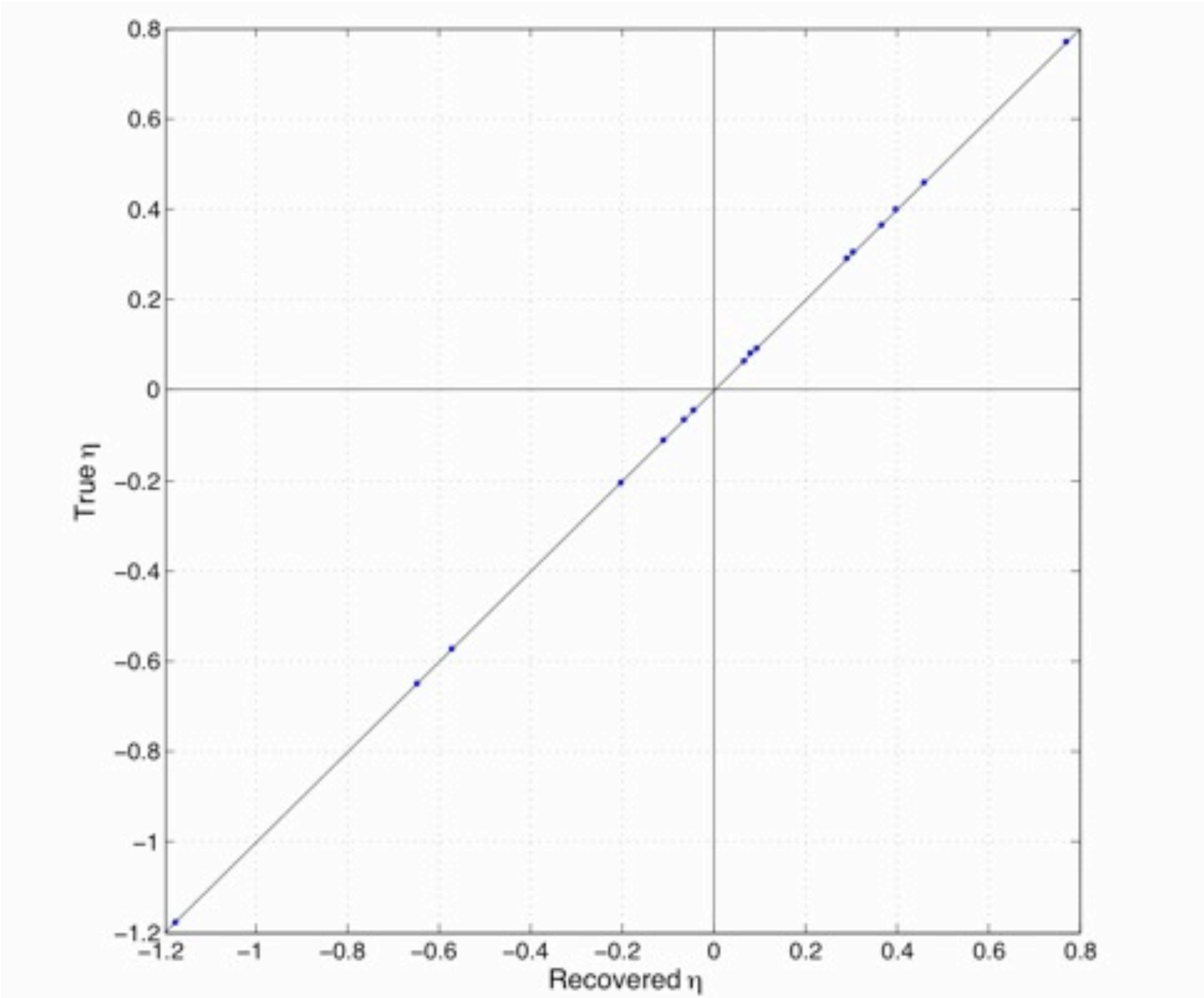
- No knowledge of absolute gain.

Imaginary part (phases):

- No knowledge of absolute phase.
- No knowledge of x-tilt.
- No knowledge of y-tilt.

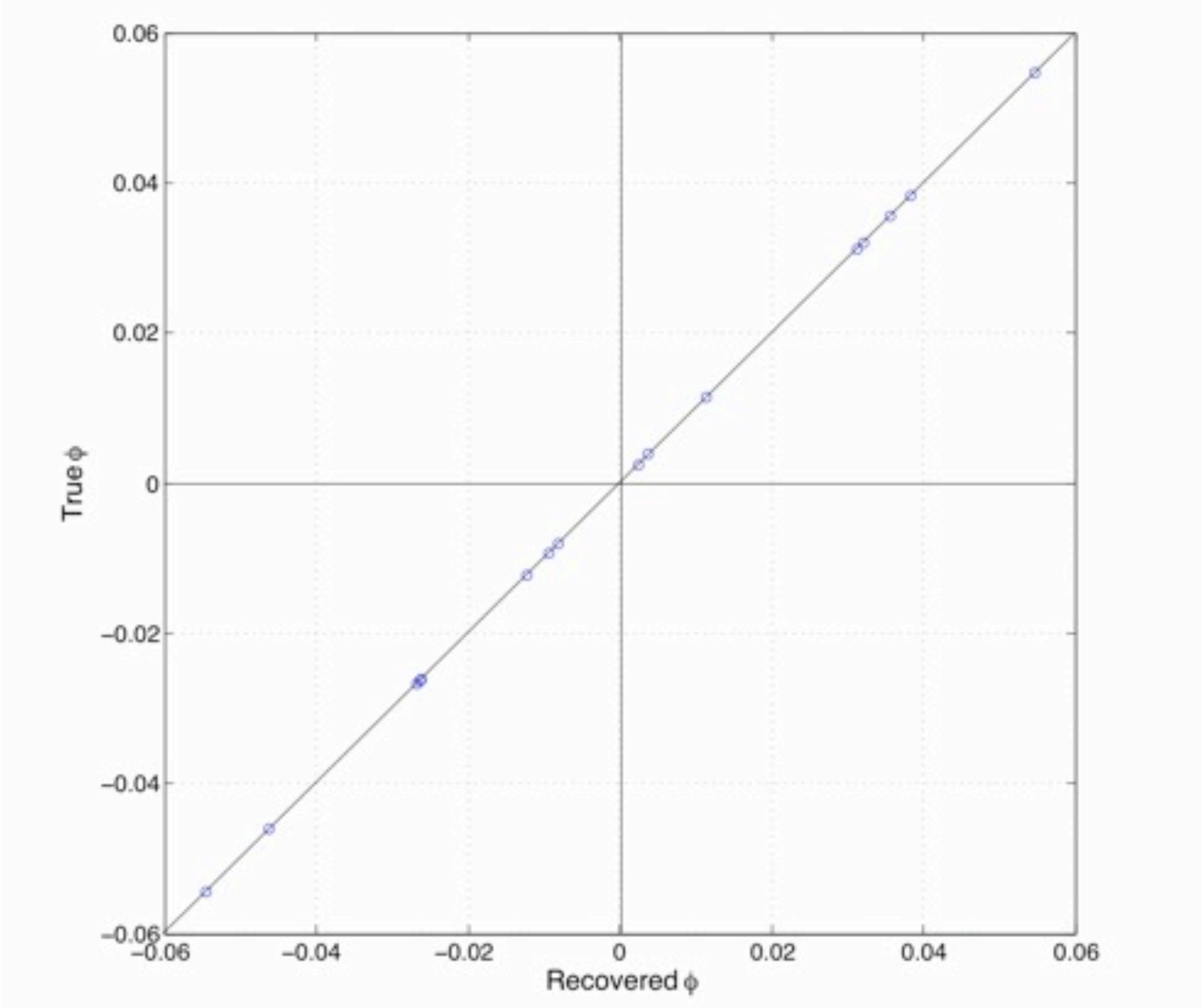
s_i = signal measured by antenna i ; g_i = gain of antenna i

; $g_i \equiv e^{\eta_i + i\varphi_i}$ corr; c_{ij} = measured corr



s_i = signal measured by antenna i ; g_i = gain of antenna i

; $g_i \equiv e^{\eta_i + i\varphi_i}$ corr; c_{ij} = measured corr



s_i = signal measured by antenna i ; g_i = gain of antenna i

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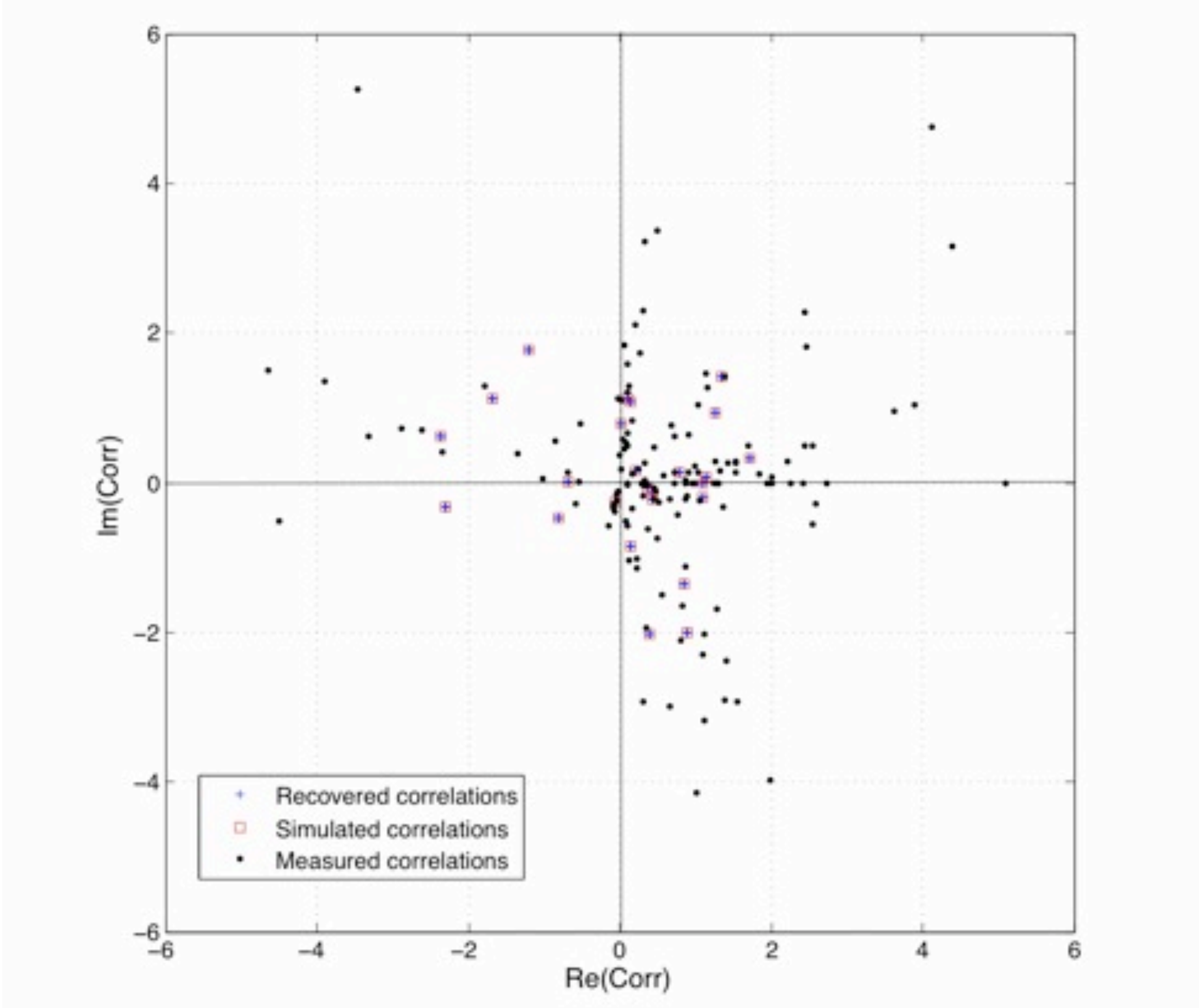
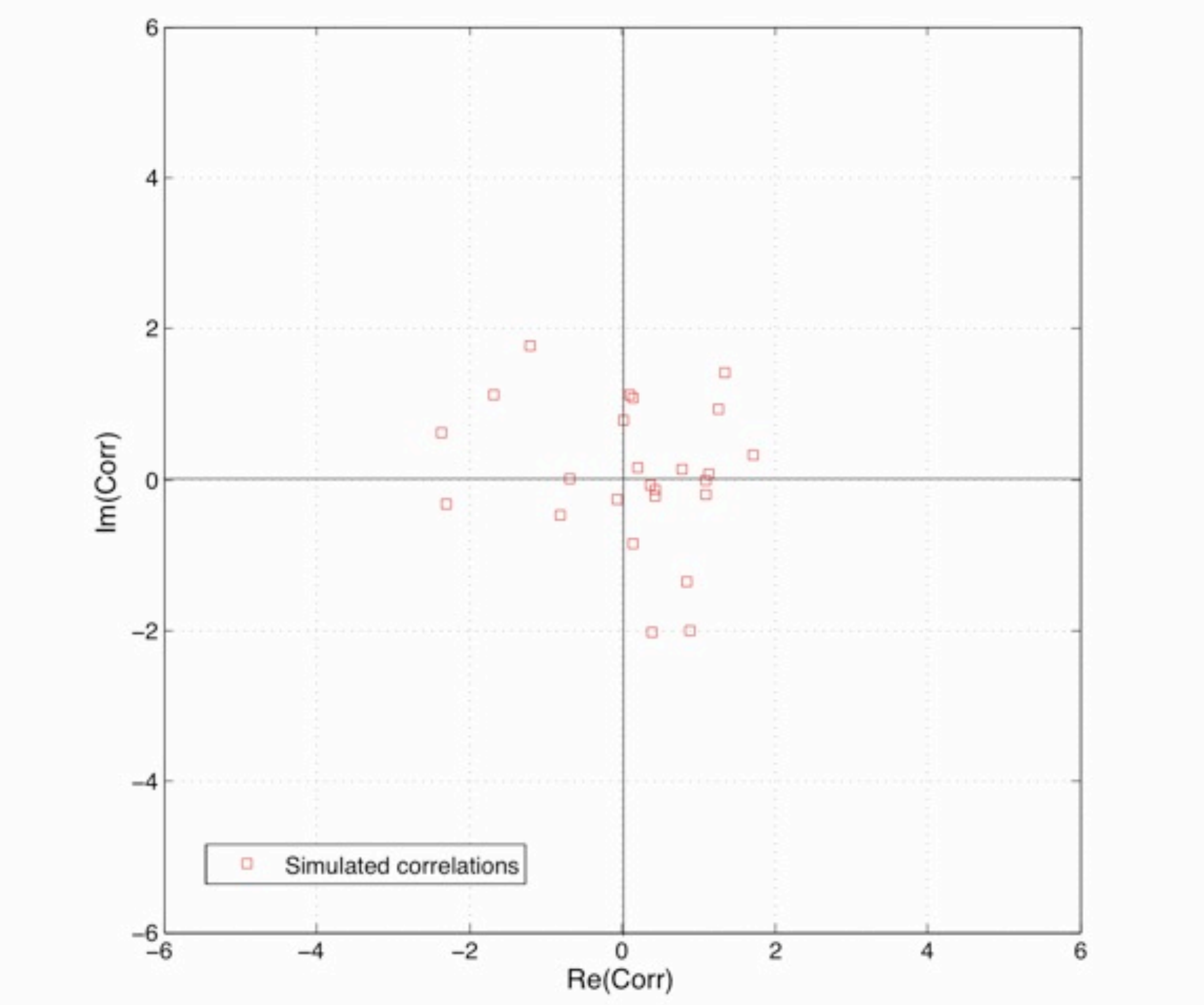
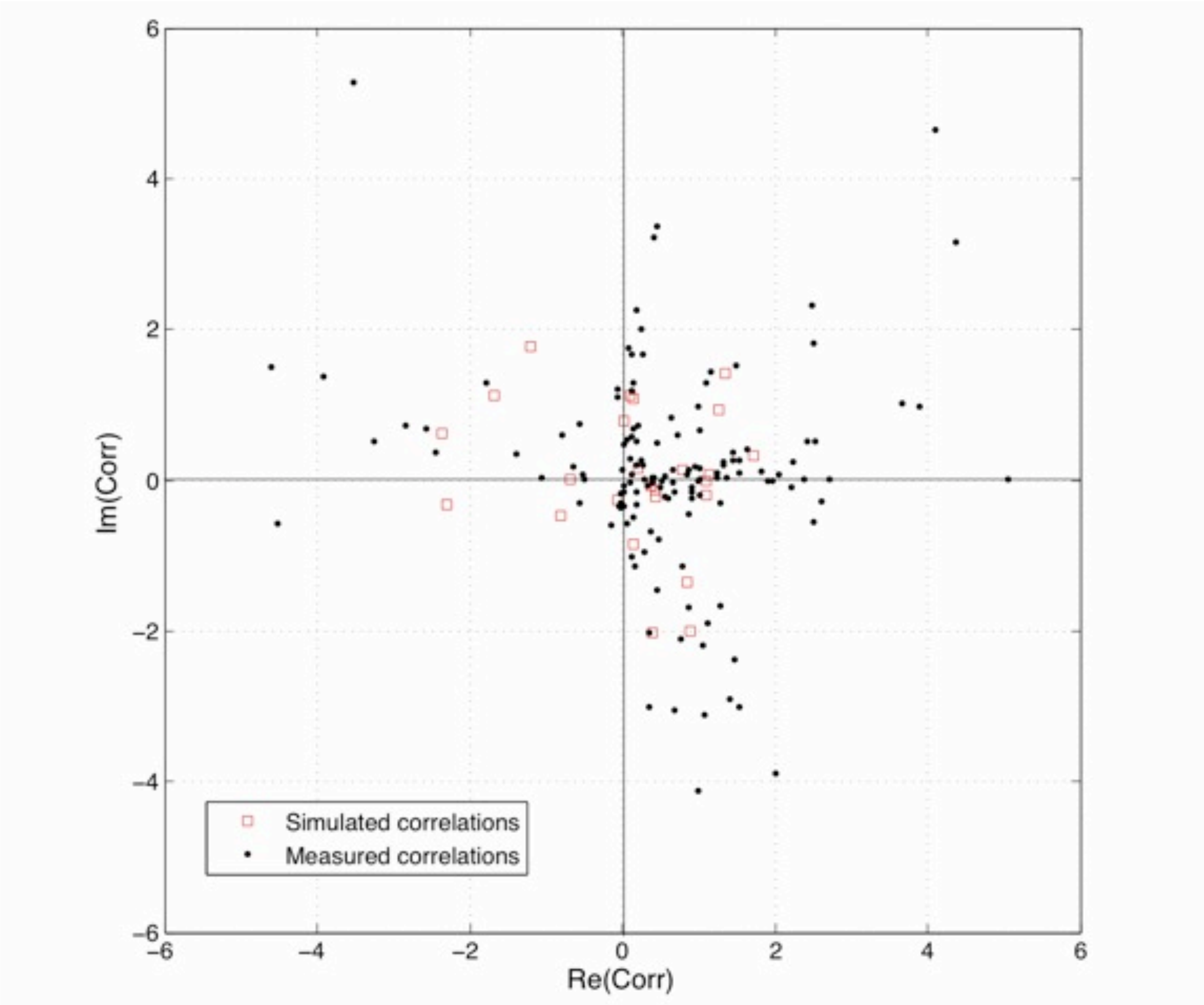


s_i = signal measured by antenna i ; g_i = gain of antenna i

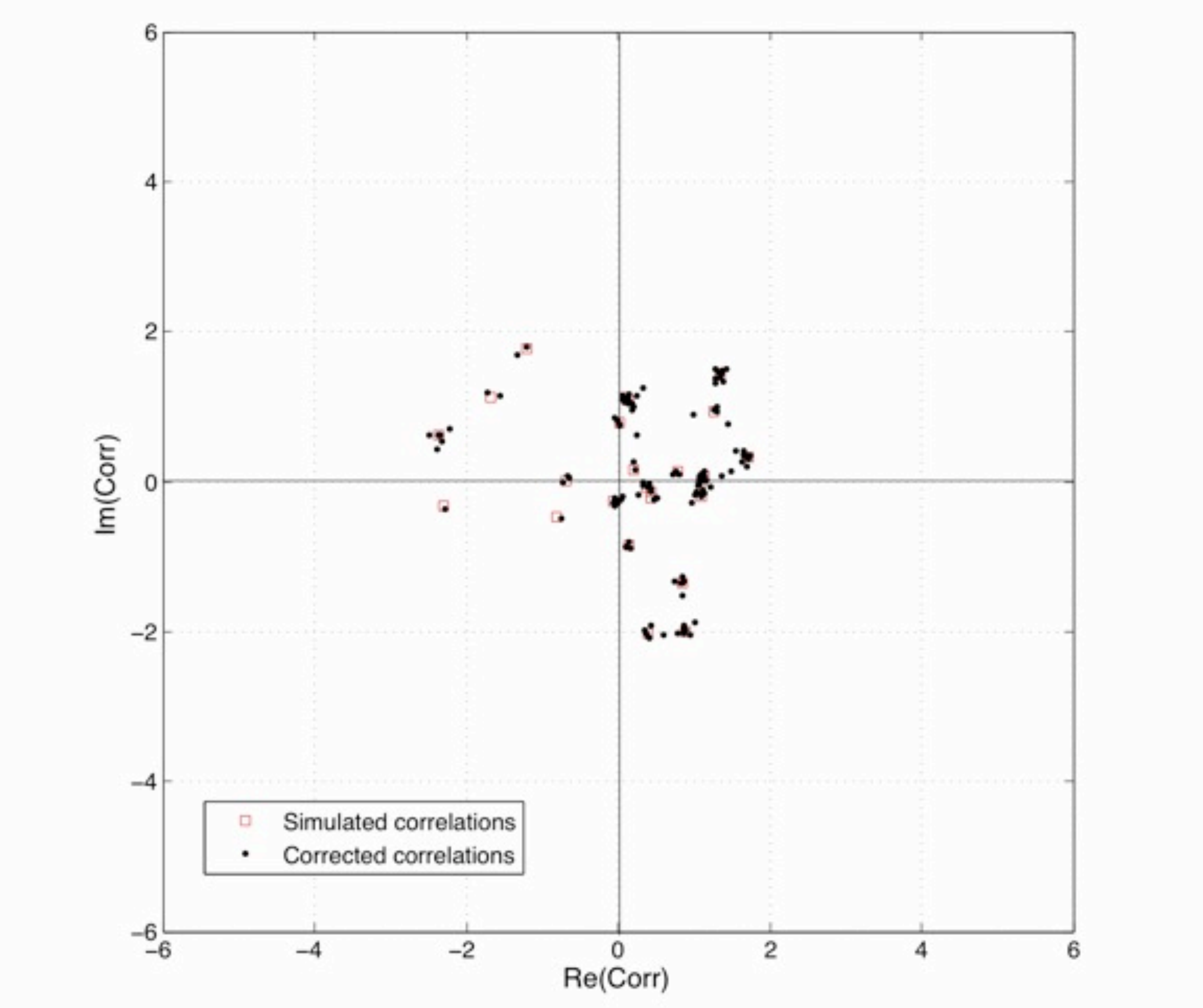
;$g_i \equiv e^{\eta_i + i\varphi_i}$ corr; c_{ij} = measured corr



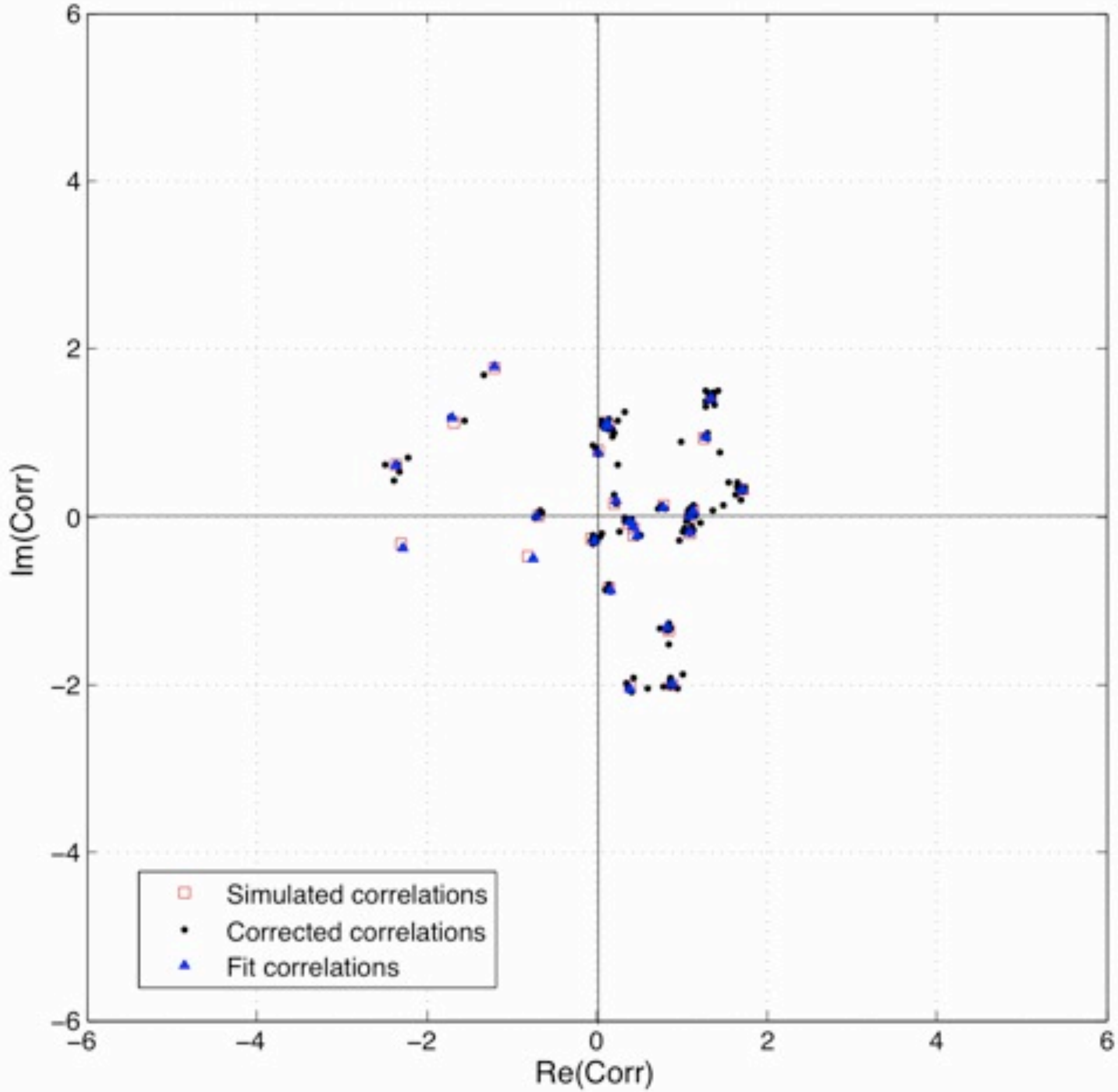
Im(Corr)
Re(Corr)
Simulated correlations
6
4
2
0
−2
−4
−6



Im(Corr)
Re(Corr)
Simulated correlations
Measured correlations
6
4
2
0
−2
−4
−6

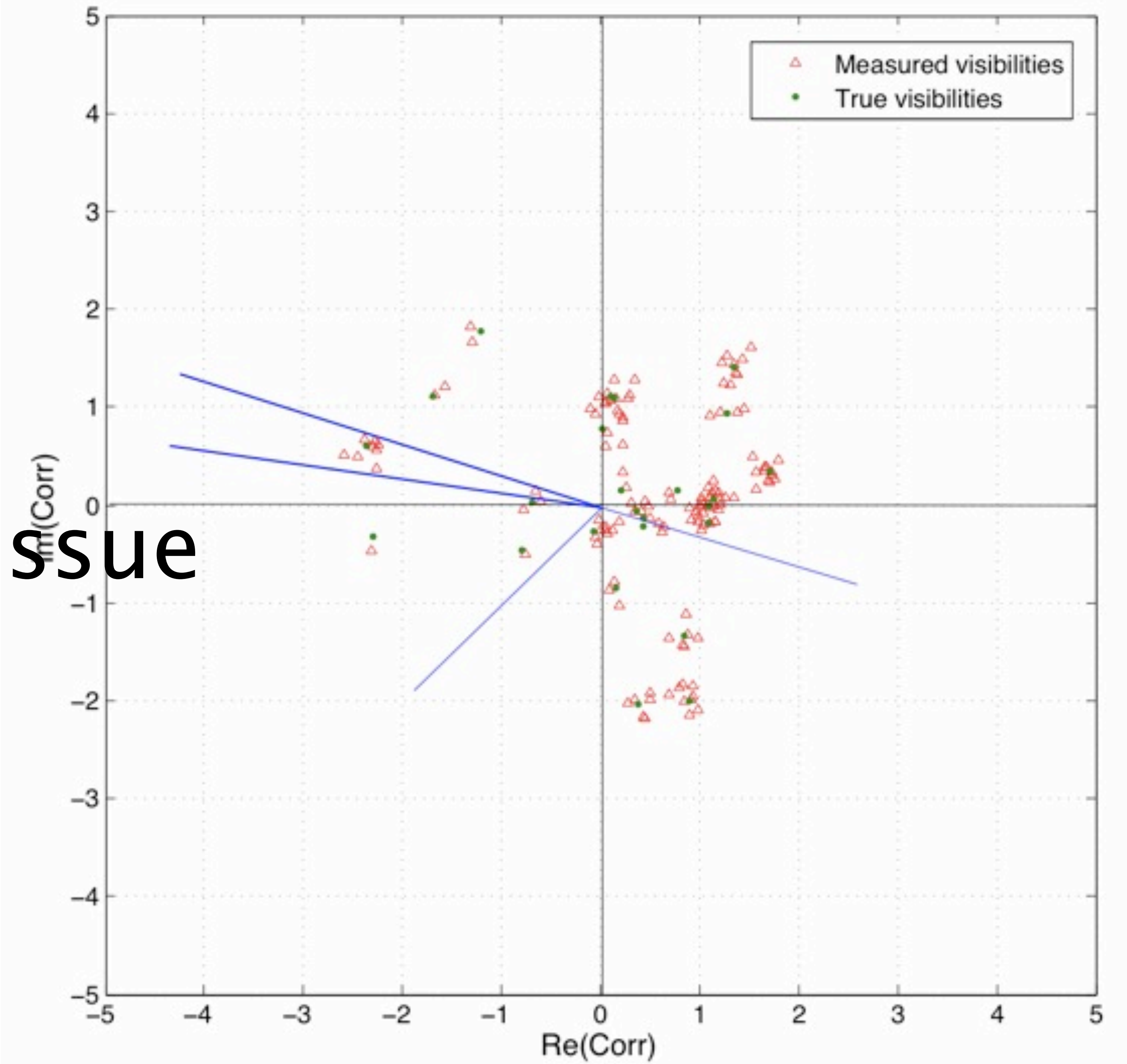


Im(Corr)
Re(Corr)
Simulated correlations
Corrected correlations



6
4
2
0
−2
−4
−6
Im(Corr)
−6
−4
−2
0
2
4
6
Re(Corr)
Simulated correlations
Corrected correlations
Fit correlations

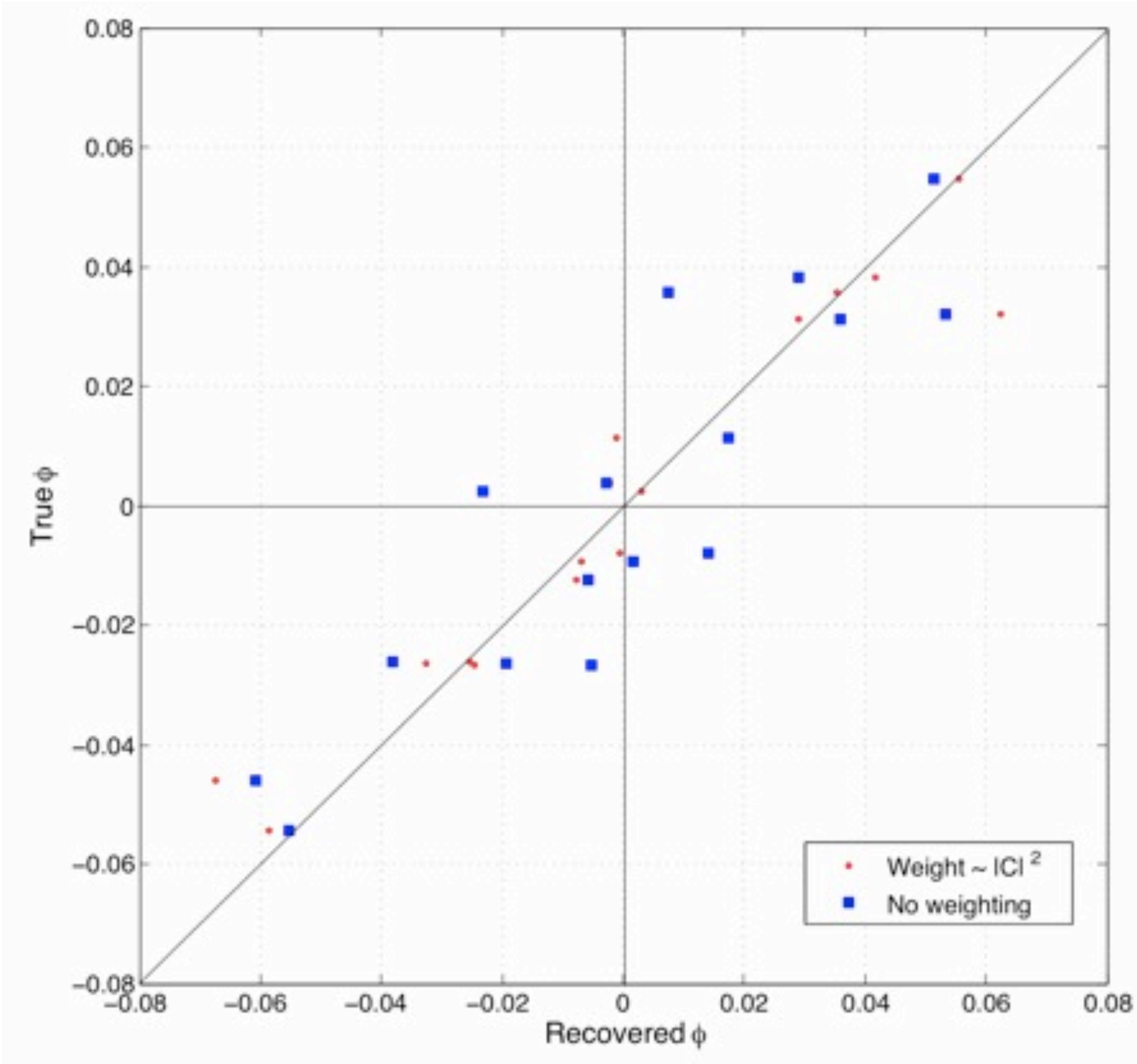
eighty Issue



Weighted Fit

$$\hat{b} = [A^t W A]^{-1} A^t W \vec{d}$$

$$W_{ij} = \delta_{ij} |c_i|^2$$



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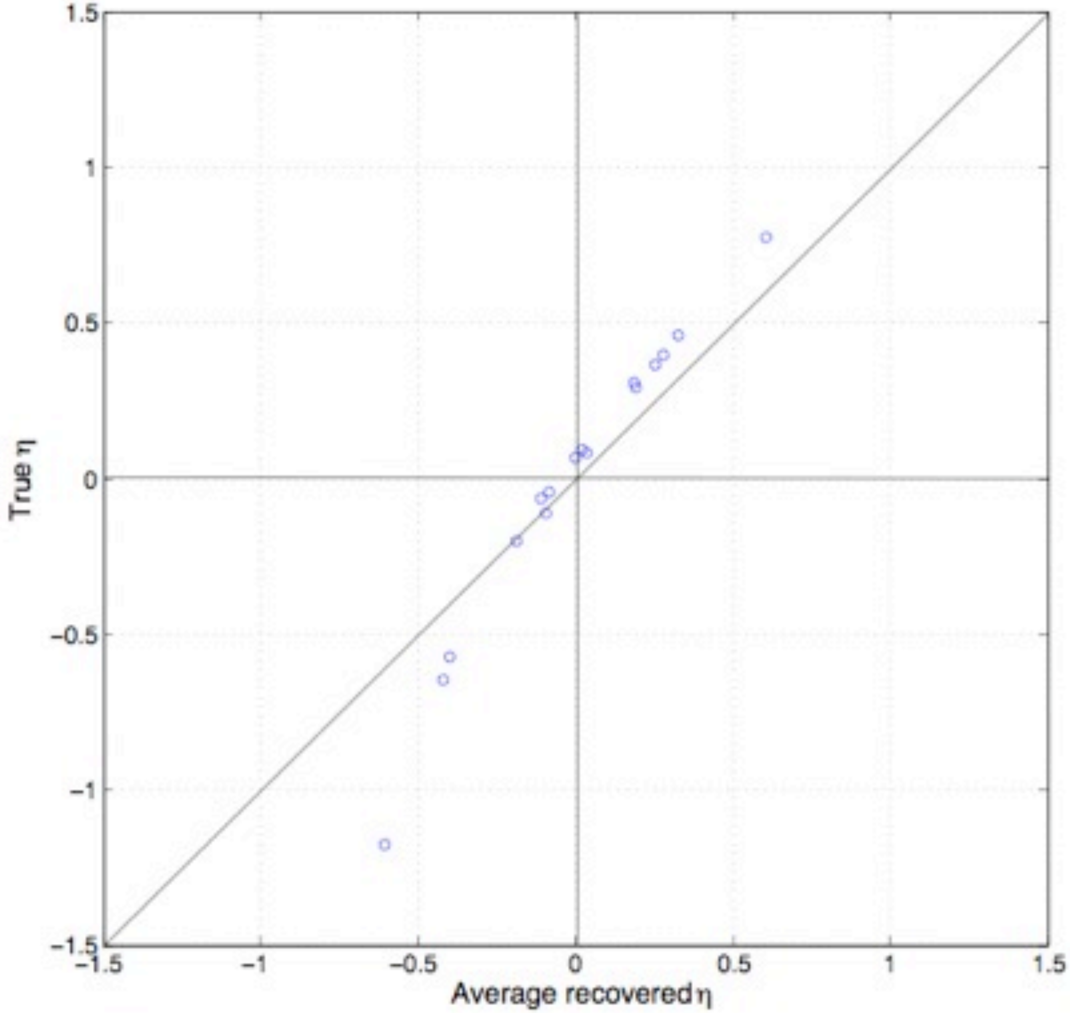
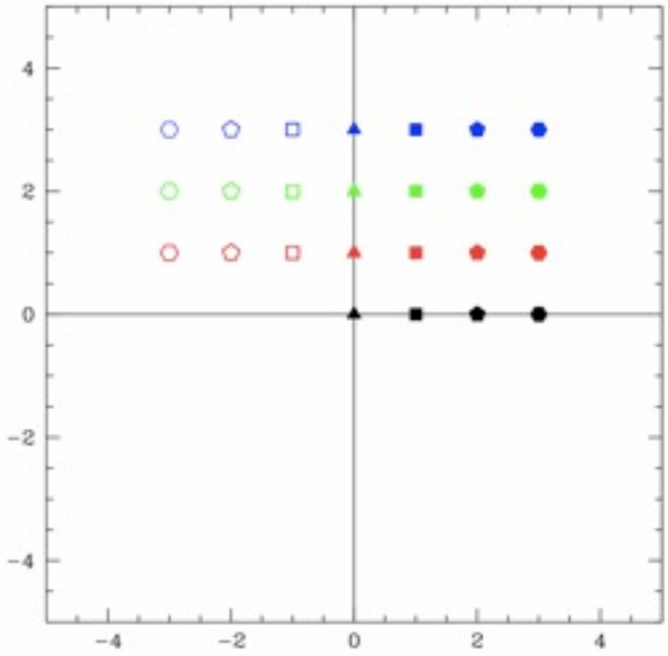
Some Bias

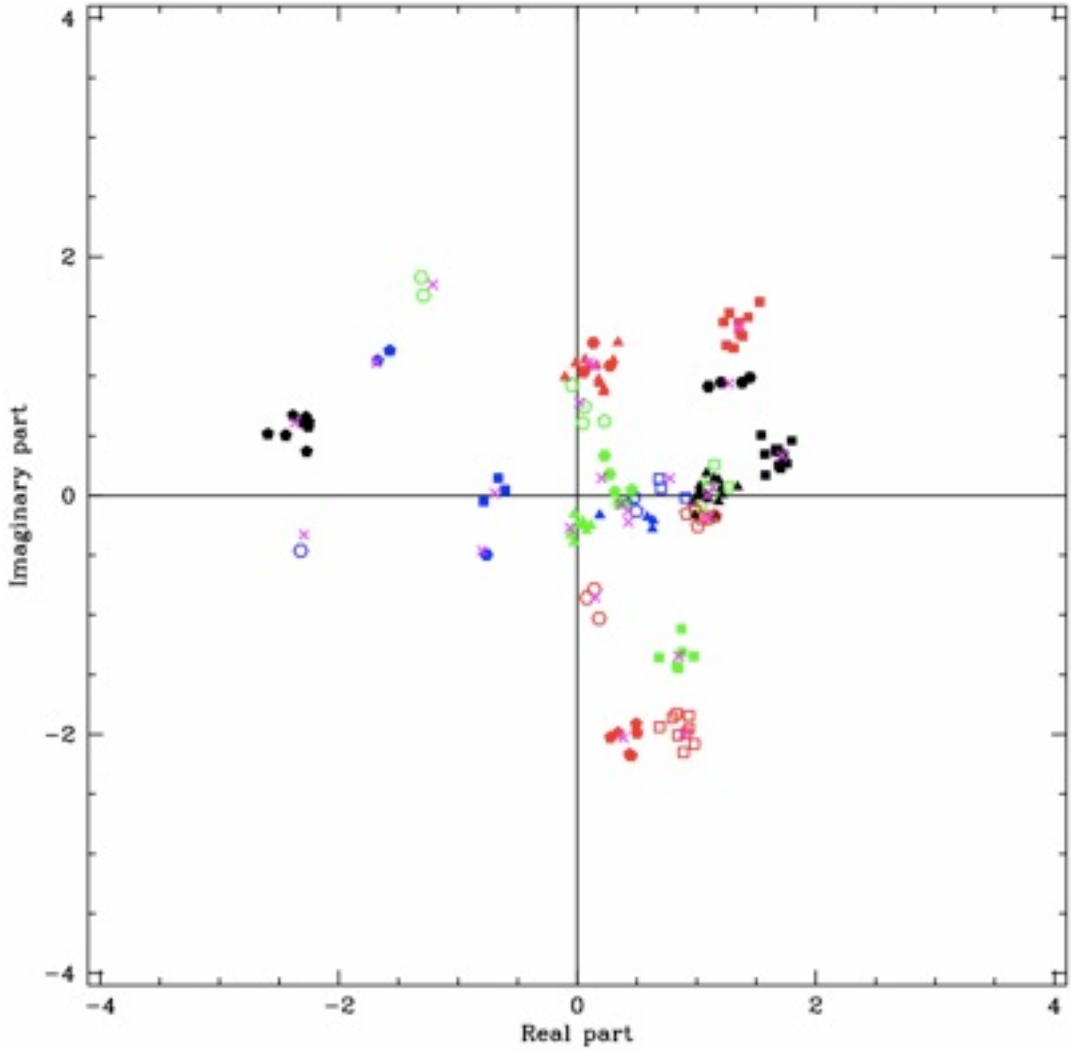


Differences Between This and Traditional Calibration

Traditional Scheme	Redundant Baselines
• Use closure phases • Amplitude (gain) calibration requires point source • Modeling may be required	• Any sky signal can be used for calibration • Redundancy provided by identical baselines



4
2
0
−2
−4
−4
−2
0
2
4



4
2
0
−2
−4
Imaginary part
−4
−2
0
2
4
Real part

Summary

- Redundant baselines provided by omniscopes can be exploited for calibration
- Proposed automatic calibration scheme works on simulated data
- Using redundant baselines allows one to sidestep certain problems with traditional techniques (like the necessity of having isolated point sources).